Square Roots & Irrational Numbers

$\sqrt{}$

The square of an integer is a PERFECT SQUARE.

Example - $1^2 = 1$ $2^2 = 4$ $3^2 = 9$

The inverse of squaring a number is finding the SQUARE ROOT. The square root radical indicates the NONNEGATIVE square root of a number. (In this class, you can assume all our square roots are positive.)

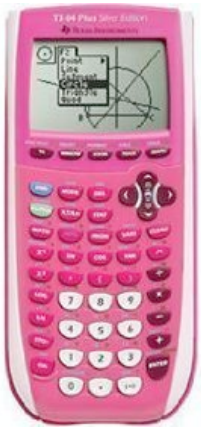
Simplifying Square Roots

Simplify each square root.

$\sqrt{64}$ $-\sqrt{64}$

$\sqrt{100}$ $-\sqrt{100}$

$\sqrt{16}$ $-\sqrt{16}$

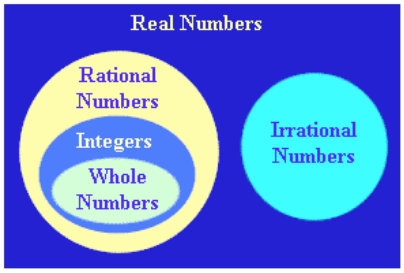


As we've learned this year, you can express a rational number as the ratio of two integers, a/b, as long as $b \neq 0$. In decimal form, a rational number either terminates or repeats.

An **IRRATIONAL NUMBER** has a decimal form that neither terminates nor repeats and cannot be written as a ratio. Can you think of a common example?

IF AN INTEGER IS NOT A PERFECT SQUARE, ITS SQUARE ROOT IS IRRATIONAL.

Together, rationals and irrationals for the real numbers. Remember this from Ch. 4?



Identifying Irrational Numbers

Use your calculator to help identify each number as rational or irrational. Justify your answers.

$\sqrt{18}$

$\sqrt{121}$

432.8

$0.121212\ldots$

$0.120120012\ldots$

π

The Pythagorean Theorem

The Pythagorean Theorem applies to right triangles. Do you remember what a right triangle is?

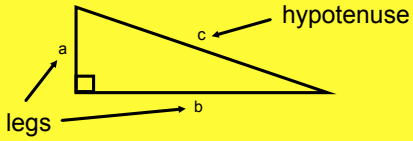
In a right triangle, the two shortest sides are called LEGS. The longest side, which is opposite the right angle, is called the HYPOTENUSE.

The Pythagorean Theorem shows how the legs and hypotenuse of a right triangle are related.

THE PYTHAGOREAN THEOREM

In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$a^2 + b^2 = c^2$



Using the Pythagorean Theorem

The lengths of two sides of a right triangle are given. Find the length of the third side.

a. legs: 3 ft. and 4 ft.

b. leg: 12 m.; hypotenuse: 15 m.

But what happens when you're dealing with a nonperfect square?

Use your calculator to approximate!

In a right triangle, the length of the hypotenuse is 15 m and the length of a leg is 8 m. What is the length of the other leg to the nearest tenth of a meter?

Now what would you do with this one?

Can you form a right triangle with the three lengths of 5 mm, 6 mm, and 10 mm?

Special Right Triangles

When we learned about Pythagorean Theorem, you were required to understand how square roots work and that taking the square root of a number is the opposite, or inverse, of squaring a number.

Today we're going to add to our knowledge of square roots by learning the rule for Multiplying Square Roots:

Key Concepts Multiplying Square Roots

For nonnegative numbers, the square root of a product equals the product of the square roots.

Arithmetic	Algebra
$\sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2}$	If $a \geq 0$ and $b \geq 0$, then $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

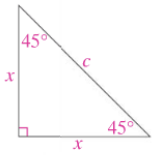
Two Types of Special Right Triangles:

1. 45°-45°-90° Triangles, and
2. 30°-60°-90° Triangles

45°-45°-90° Triangles

The rule for Multiplying Square Roots is especially useful with an isosceles right triangle, which is also known by its angle measures as a 45°-45°-90° triangle. You can use the rule to relate the lengths of the sides and the hypotenuse in such a triangle. Consider the following:

$c^2 = a^2 + b^2$	Use the Pythagorean Theorem.
$c^2 = x^2 + x^2$	Replace *a* and *b* with *x*.
$c^2 = 2x^2$	Simplify.
$c = \sqrt{2x^2}$	Find the square root.
$c = \sqrt{2} \cdot \sqrt{x^2}$	Use the rule for Multiplying Square Roots.
$c = \sqrt{2} \cdot x$, or $x\sqrt{2}$	Simplify.



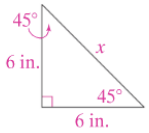
Key Concepts 45°-45°-90° Triangles

In a 45°-45°-90° triangle, the legs are congruent and the length of the hypotenuse is the length of a leg times $\sqrt{2}$.

$$\text{hypotenuse} = \text{leg} \cdot \sqrt{2}$$

You can use this relationship to find the missing length of a hypotenuse.

Find the length of the hypotenuse in the triangle at the right.



$\text{hypotenuse} = \text{leg} \cdot \sqrt{2}$	**Use the 45°-45°-90° relationship.**
$x = 6 \cdot \sqrt{2}$	**The length of the leg is 6.**
≈ 8.5	**Use a calculator.**

The length of the hypotenuse is about 8.5 in.

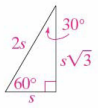
Now you try one:

The length of each leg of an isosceles right triangle is 4.2 cm. Find the length of the hypotenuse. Round to the nearest tenth.

30°-60°-90° Triangles

Key Concepts 30°-60°-90° Triangle

In a 30°-60°-90° triangle, the length of the hypotenuse is 2 times the length of the shorter leg. The length of the longer leg is the length of the shorter leg times $\sqrt{3}$.

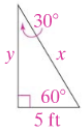


$$\text{hypotenuse} = 2 \cdot \text{shorter leg}$$

$$\text{longer leg} = \text{shorter leg} \cdot \sqrt{3}$$

You can use this relationship to find missing lengths:

Find the missing lengths in the triangle.



$\text{hypotenuse} = 2 \cdot \text{shorter leg}$

$x = 2 \cdot 5$ **The length of the shorter leg is 5.**

$x = 10$ **Simplify.**

$\text{longer leg} = \text{shorter leg} \cdot \sqrt{3}$

$y = 5 \cdot \sqrt{3}$ **The length of the shorter leg is 5.**

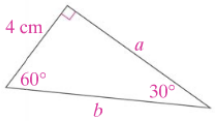
$y \approx 8.7$ **Use a calculator.**

The length of the hypotenuse is 10 ft, and the length of the longer leg is about 8.7 ft.

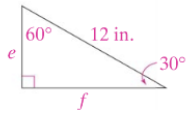
Let's try a few...

Find the missing lengths in each 30°-60°-90° triangle:

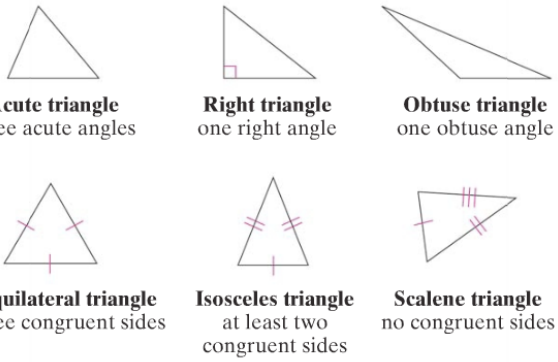
a.



b.



Do you remember the different types of triangles? You can classify triangles based on angle measures OR side lengths:



Acute triangle
three acute angles

Right triangle
one right angle

Obtuse triangle
one obtuse angle

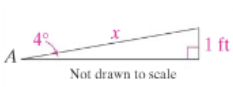
Equilateral triangle
three congruent sides

Isosceles triangle
at least two congruent sides

Scalene triangle
no congruent sides

Sine, Cosine, & Tangent Ratios

The ratio of the lengths of two sides of a right triangle is called a TRIGONOMETRIC RATIO. These ratios can help you find missing measures of triangles when you don't have enough information to use Pythagorean Theorem. Check out this example:



How would you solve this problem? Could you use Pythag?

When you don't have enough info to use Pythag, you can use...

SOH CAH TOA!!!

Who the, what the, huh?!?

SOH CAH TOA is an acronym or way to remember the three trigonometric ratios of sine, cosine, and tangent.

Trigonometric Ratios

$$\text{sine } \angle A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}}$$

$$\text{cosine } \angle A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}}$$

$$\text{tangent } \angle A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A}$$

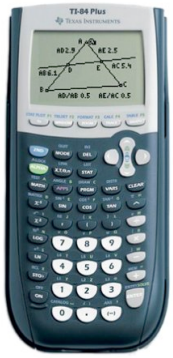
These look and sound much more complicated than they are. It's easier to remember them using SOH CAH TOA:

Sine = $\frac{\text{Opposite}}{\text{Hypotenuse}}$ Cosine = $\frac{\text{Adjacent}}{\text{Hypotenuse}}$ Tangent = $\frac{\text{Opposite}}{\text{Adjacent}}$

Evaluating the Ratios

So how in the world do I figure out what sine (60°) is anyway?!?

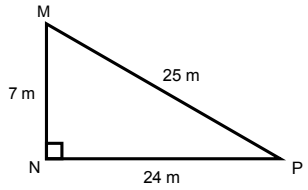
It's easy! Just use your calculator!



Try these. Round to four decimal places.

cos 20°
sin 41°
sin 88°
tan 64°
tan 8°
cos 53°

Finding the Ratios...the simple stuff!

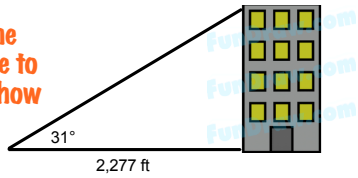


Give the ratios for each:

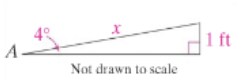
sin ∠P =
cosine ∠P =
tangent ∠P =
sine ∠M =
cosine ∠M =
tangent ∠M =

Word Problem Applications...the not so simple stuff!

A surveyor standing 2,277 ft from the base of a building measured a 31° angle to the topmost point. To the nearest ft, how tall is the building?



So let's go back to our original example:



Ask yourself, what info do I have?
What info am I looking for?

Ramps **What is the length of the wheelchair ramp at the left?**

You know the angle and the side opposite the angle. You want to find x, the length of the hypotenuse.

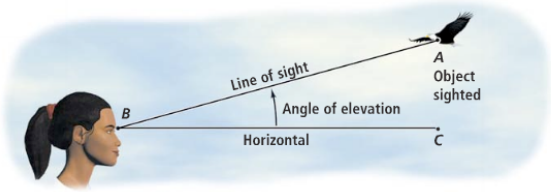
$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$	**Use the sine ratio.**
$\sin 4^\circ = \frac{1}{x}$	**Substitute 4° for the angle and 1 for the side opposite.**
$x(\sin 4^\circ) = 1$	**Multiply each side by x.**
$x = \frac{1}{\sin 4^\circ}$	**Divide each side by sin 4°.**
$x \approx 14.3$	**Use a calculator.**

The ramp is about 14.3 ft long.

Angles of Elevation & Depression

Civil engineers and navigators use the terms angle of elevation and angle of depression to describe the angles at which they see things.

An ANGLE OF ELEVATION is formed by a horizontal line and a line of sight above it. In other words, this type of angle is formed when you LOOK UP AT SOMETHING.



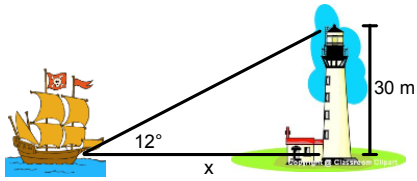
$\angle ABC$ is an angle of elevation.

You can use angles of elevation and depression in conjunction with SOH CAH TOA to calculate distances through indirect measurement.

Using Angles of Elevation

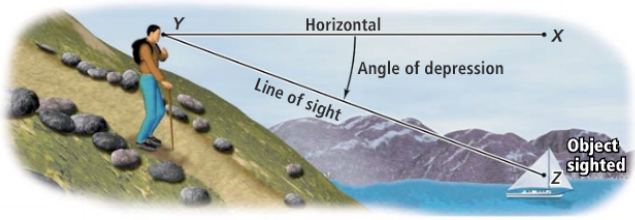
The angle of elevation from a ship to the top of a lighthouse is 12°. The lighthouse is known to be 30 m tall. How far is the ship from the base of the lighthouse?

(*HINT: one of the toughest things about these problems is knowing what you are being asked to find. Drawing a diagram for these types of problems is KEY!)



Now that you have a visualization of the triangle you're dealing with, how could you use SOH CAH TOA to find the missing measure?

An ANGLE OF DEPRESSION is formed by a horizontal line and a line of sight below it. In other words, this type of angle is formed when YOU LOOK DOWN AT SOMETHING.



$\angle XYZ$ is an angle of depression.

You can use SOH CAH TOA to calculate distances here too!

(*HINT: with an angle of elevation, the object sighted is ELEVATED, or above the horizontal line. With an angle of depression, the object is DEPRESSED, or below the horizontal line.)

Using Angles of Depression

A group of people in a hang-gliding class are standing on top of a cliff 70 m high. They spot a hang glider landing on the beach below them. The angle of depression from the top of the cliff to the hang glider is 72°. How far is the hang glider from the base of the cliff?